Math 110
Winter 2021
Lecture 15

Ch. 7 Estimating Parameters Sec 23 & 24

To estimate Parameters, we find range of values for the estimation.

Range of values is called Confidence Interval

Every Confidence interval comes with Confidence level.

Here are Some Common Conf. level:

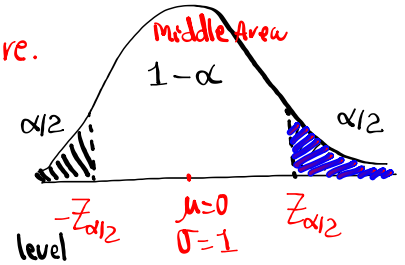
90%, 95%, 98%, 99%

when C-level not given

$\Rightarrow$ use 95% C-level.

Confidence level is the middle area of the graph of Prob. dist.

The values that separate the middle area from the rest are called Critical Values such as $z_{\alpha/2}$, $t_{\alpha/2}$, and more.



α → Alpha

$0 < \alpha < 1$

α is called Significance level

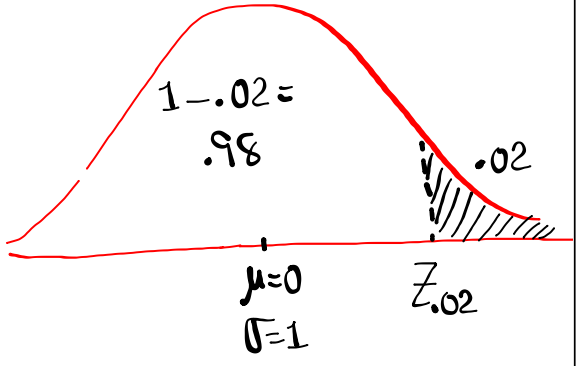
when C-level not given ⇒ use 95%

when α not given ⇒ use $\alpha = .05$

$\alpha/2$ is the right-area

How to find Critical Value $z_{\alpha/2}$:

Find $z_{.02}$ → Right-area



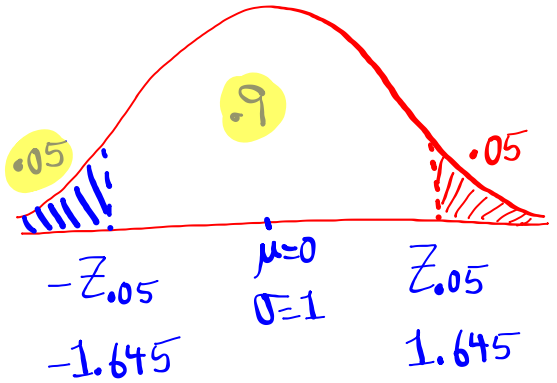
$z_{.02} = \text{invNorm}(.98, 0, 1)$

$= \boxed{2.054}$

Find $z_{\alpha/2}$ for $\alpha = .1$

$1 - \alpha = 1 - .1 = .9$

$\alpha/2 = .1/2 = .05$ Right-area



$z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

Find $z_{\alpha/2}$ for 99% Confidence level.

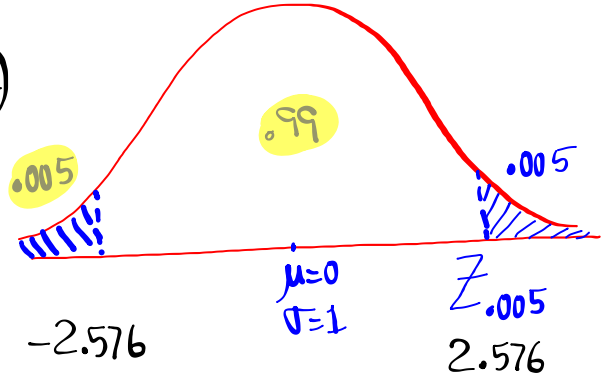
middle Area

$1 - .99 = .01$

$.01/2 = .005$

$z_{\alpha/2} = \text{invNorm}(.995, 0, 1)$

$= \boxed{2.576}$



Estimating Population Proportion p

Final Answer $\quad < p <$ Confidence Interval

Format $\quad \hat{p} - E < p < \hat{p} + E$

$\hat{p}$ p-hat, Sample Proportion $\quad \hat{p} = \frac{x}{n}$

$\hat{q}$ q-hat, $\quad \hat{q} = 1 - \hat{p}$

E Margin of error $\quad E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

I surveyed 400 students, 300 of them had iphone.

$n = 400$

$x = 300$

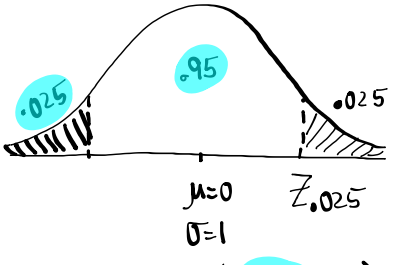
$\hat{p} = \frac{x}{n} = \frac{300}{400} = .75$ $\quad \boxed{\hat{p} = .75}$ $\quad \boxed{\hat{q} = .25}$

Find 95% Confidence interval for the prop. of all Students that have iPhone.

C-level : .95

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.960 \cdot \sqrt{\frac{(.75)(.25)}{400}}$

$= \boxed{.042}$



$\hat{p} - E < p < \hat{p} + E$

$.75 - .04 < p < .75 + .04$

$\boxed{.71 < p < .79}$

$z_{.025} = \text{invNorm}(.975, 0, 1) = 1.960$

Now using TI:

STAT → TESTS ↓ 1-PropZInt

$E = \frac{.79 - .71}{2} = .04$

$x = 300$

$n = 400$

$.71 < P < .79$

C-level: .95

Calculate

$\hat{P} = \frac{.79 + .71}{2} = .75$

Point-estimate

I surveyed 750 students, 8% of them were left-handed.

1) How many were left-handed? $n = 750$

8% of 750 = .08(750) ⇒ $x = 60$

If decimal ⇒ Round-up

2) Find 99% Confidence interval for the prop. of all students that are left-handed.

C-level: .99

1-PropZInt

$x = 60$

$n = 750$

C-level: .99

Calculate

$.054 < P < .106$

$E = \frac{.106 - .054}{2} = .026$

$\hat{P} = \frac{.106 + .054}{2} = .08$

In a survey of 326 students, 42% of them were in favor of Zoom lectures.

1) How many were in favor of Zoom lectures?

42% of $326 = .42(326) = 136.92$ $\boxed{x = 137}$

If decimal $\Rightarrow$ Round-up

2) Find Confidence interval for the prop. of all Students in favor of Zoom lectures.

NO C-level $\Rightarrow$ use .95

$\boxed{.367 < p < .474}$

1-PropZInt
$x = 137$
$n = 326$ C-level: .95

3) $E = \dfrac{.474 - .367}{2} = \boxed{.054}$

4) $\hat{p} = \dfrac{.474 + .367}{2} = \boxed{.421}$

Estimating Population Mean μ

Final Answer $< \mu <$

Format $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$ Sample Mean, Point-estimate

E Margin of error.

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	
STAT TESTS ZInterval inpt: Stats	

I surveyed 48 students, their mean age was 31.5 yrs. $n=48$ $\bar{x}=31.5$

It is known that standard deviation of ages of all students is 7.2 yrs. $\sigma=7.2$

Find 88% Confidence interval for mean age of all students.

C-level: .88

$\bar{x}-E < \mu < \bar{x}+E$

$31.5-1.6 < \mu < 31.5+1.6$

$29.9 < \mu < 33.1$

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 1.555 \cdot \frac{7.2}{\sqrt{48}} = 1.616$

ZInterval

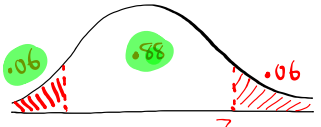
inpt: STATS

$\sigma=7.2$ $E=\frac{33.1-29.9}{2}=1.6$

$\bar{x}=31.5$ $\bar{x}=\frac{33.1+29.9}{2}=31.5$

$n=48$

C-level: .88



$z_{.06} = \text{invNorm}(.94, 0, 1) = 1.555$

In a survey of 40 nurses in So. Cal., their mean Salary was \$6250/mo. $n=40$ $\bar{x}=6250$

Dept. of health Servies has reported that Standard deviation of Salaries of all nurses is \$475/mo. $\sigma=475$

Find 98% Conf. interval for the mean Salary of all nurses in So. Cali.

C-level: .98

$6075 < \mu < 6425$

σ known $\Rightarrow$ ZInterval

Inpt: STATS

$\sigma=475$ $\bar{x}=6250$ $n=40$ C-level: .98

$E=\frac{6425-6075}{2}=175$ $\bar{x}=\frac{6425+6075}{2}=6250$

I randomly Selected 30 exams. Here are the Scores:

94	75	88	65	59
100	70	80	90	92
97	99	83	79	78
78	54	68	93	87
70	82	60	90	93
72	89	91	81	77

$76 < \mu < 86$

$E = \frac{86-76}{2} = 5$

$\bar{x} = \frac{86+76}{2} = 81$

Clear all lists

Store in L1

Find $\bar{x}$, Round to whole # $\bar{x}=81$

Assume $\sigma = 15$, Find Confidence interval for the mean of all exams.

σ Known

ZInterval

inpt: STATS

$\sigma=15$, $\bar{x}=81$, $n=30$

C-level: .95

Estimating Population Mean μ

Final Answer $< \mu <$

Format $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$ Sample Mean, Point-estimate

E Margin of error.

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
	$df = n-1$
STAT TESTS ZInterval inpt: Stats	STAT TESTS TInterval inpt: Stats

Given: $n=20$, $\bar{x}=82$, $S=10$, C-level: .9

Find confidence interval for μ.

σ known $\Rightarrow$ ZInterval

σ unknown $\Rightarrow$ TInterval

STAT
TESTS
TInterval
inpt: STATS

$\bar{x}=82$
$S=10$
$n=20$
C-level: .9
Calculate

$78.1<\mu<85.9$

$E=\frac{85.9-78.1}{2}=3.9$ $\bar{x}=\frac{85.9+78.1}{2}=82$

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12 police officers had a mean age of 49.5 with standard deviation of 9.2 yrs.

Find 99% Confidence interval for the mean age of all police officers.

$n=12$ $\bar{x}=49.5$ $S=9.2$

C-level: .99

$41.3<\mu<57.7$

TInterval

$E=\frac{57.7-41.3}{2}=8.2$

$\bar{x}=\frac{57.7+41.3}{2}=49.5$

σ known $\Rightarrow$ ZInterval

σ unknown $\Rightarrow$ TInterval

WORK on SG 23 & 24